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A NOTE ON THE MICROMECHANICS OF PLASTIC YIELD OF POROUS SOLIDS

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Abstract. Experimental evidence suggests that, for materials exhibiting no appreciable work hardening and containing no more than approximately 20% volume fraction of pores, the macroscopic strain at which yield occurs is nearly constant, with a tendency to increase slightly as porosity increases. The present work shows that both observations - approximate constancy of strain at yield and its tendency to increase with porosity - have a relatively simple micromechanical explanation.

1. Introduction. Two primary thrusts in the plasticity of porous solids have been in (1) small overall strains, allowing for the possibility of large microscale deformations, with the goal being to express a macroscopic yield surface in stress space in terms of porosity, for cases when such a surface can be clearly identified and (2) the issue of void growth and coalescence at much larger overall strains. Problem (2) has been intensely investigated in recent years. A large number of these analyses stem from the well-known analysis of Gurson (1977). A particularly lucid analysis of Gurson's model, and its early extensions, can be found in Mear (1990), while more recent elucidations of the model can be found in Pardoen and Hutchinson (2000).

The present paper focuses on Problem (1), and utilizes the following earlier findings. Experimental evidence (Wang et al. 1996, ; Da Silva and Ramesh 1997a, 1997b; Kee et al. 1998) suggests that, for materials with no appreciable work hardening in bulk (at zero porosity) and containing no more than approximately 20 % volume fraction of pores, the macroscopic strain at which yield occurs is relatively insensitive to the pore volume fraction; more precisely, the yield strain has the tendency to increase slightly as porosity increases (Figure 1). This clearly identifiable yield point separates the stage of a more or less linear stress-strain relation from an almost perfectly plastic flow.

Based on these observations, a micromechanical model was recently suggested for such materials by Sevostianov and Kachanov (2001). It explains linearity of the stress-strain relation, that holds almost up to the yield point, by hypothesizing that "pockets" of local plasticity forming near pore boundaries remain contained and well embedded within the predominantly elastic field (somewhat similar to the concept

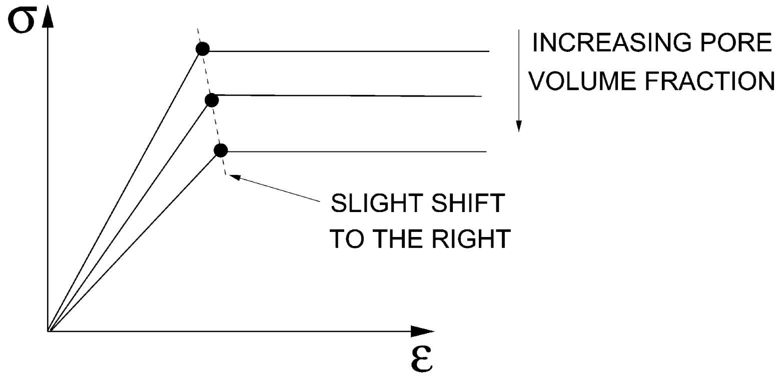

Figure 1: Idealized stress-strain curve.

of small plastic zones near crack tips). This hypothesis was confirmed by earlier microscale simulations of Zohdi et al. (2002). These local plastic "pockets" blunt the stress concentrations to such an extent that further loading produces only a very limited growth of their size. As loading increases, a transition occurs in a relatively narrow interval of stresses, that can be idealized as the yield stress, to an almost fully plasticized matrix, with nearly uniform field of the stress deviator. By using the constancy of strain at yield, the mentioned work derives the yield condition in stresses in terms of the porous space geometry. This scenario differs from the highly non-uniform deviator field for a hardening material. This difference seems natural, since stress "blunting" in a strain hardening material is substantially less pronounced, and stress concentrations at pore boundaries may lead to localizations, as the applied loads increase.

In the present work, we provide a relatively simple micromechanical explanation can be given not only for approximate constancy of the yield strain, but also for the second order effect - a slight increase of the yield strain as porosity increases, which is in agreement with cited experimental data and previous numerical simulations.

2. Dependency of yield strain on pore volume fraction. Since the yield point in poroplastic materials is somewhat "blurred", we define it as the point of intersection of the line of the horizontal "plateau" and the elastic straight line so that for pure shear loading (at infinitesimal strains), for example, $\epsilon_y \stackrel{\text{def}}{=} \tau_y/2\mu$, where τ_y is the shear stress at yield and μ is the elastic shear modulus (Figure [1]). The approximate constancy of strain at yield, with a slight tendency to increase with porosity, means that $\epsilon_y^* \stackrel{\text{def}}{=} \tau_y^*/2\mu^* \approx \epsilon_y^0 \stackrel{\text{def}}{=} \tau_y^0/2\mu^0$ or, more precisely, $\epsilon_y^* \geq \epsilon_y^0$, where index "0" and $*$ refer to the quantities for the bulk materials (zero porosity) and the considered porous material, respectively. This implies that

$$\frac{\mu^*}{\mu_0} \approx \frac{\tau_y^*}{\tau_y^0}, \tag{1}$$

or, more precisely, $\mu^*/\mu_0 \leq \tau_y^*/\tau_y^0$. We now show that both observed tendencies-approximate constancy of ϵ_y^* and its tendency to slightly increase with porosity-are consistent with relatively simple micromechanical considerations.

To estimate τ_y/τ_y^0, we assume a fully, or almost fully, plasticized state at yield. This assumption is also consistent with studies of plastic percolation in GASAR materials (Kee et al. 1998) where percolation occurred when over 80 % of the material was plasticized, and the microscale simulations of Zohdi et al. (2002). We emphasize that the hypothesis applies only to materials that experience negligible hardening in bulk (at zero porosity). Consequently, by volumetrically averaging over the volume one obtains

$$\tau_y^* = (1 - v_p)\tau_y^0, \tag{2}$$

where v_p is the volume fraction of pores (porosity).

The observed tendencies for the yield strain ϵ_y^* imply that $\mu^*/\mu_0 \approx 1 - v_p$ or, more precisely, $\mu^*/\mu^0 \leq 1 - v_p$. Estimation of μ^*/μ_0 belongs to the problem of effective elastic properties of a material with pores. In the case when the pores are spherical (results remain sufficiently accurate for moderately non-spherical randomly oriented pores, Kachanov et al., 1994), in the small porosity limit,

$$\frac{\mu^*}{\mu_0} = 1 - \frac{15(1 - \nu_0)}{7 - 5\nu_0} v_p, \tag{3}$$

where ν_0 is the Poisson's ratio of the bulk material. In this limit, therefore, the observed tendencies require that the ratio

$$\frac{1 - v_p}{1 - \frac{15(1-\nu_0)}{7-5\nu_0} v_p} \tag{4}$$

is close to unity, with a slight tendency to increase with v_p. Indeed, as v_p increases from 0 to 0.1, this ratio changes from 1 to about 1.1. At porosities of the order of 0.2, result (3) loses accuracy, and we use the upper Hashin-Shtrikman bound (the lower one is zero for pores); see Hashin and Shtrikman (1962, 1963) for further analysis. For microstructures comprised of a continuous hard matrix surrounding soft particles, it is well-known that the Hashin-Shtrikman upper bound is quite accurate, while for microstructures comprised of a continuous soft matrix surrounding hard particles, the Hashin-Shtrikman lower bound is appropriate (Hashin 1983). It is valid for all pore shapes and has the form

$$\frac{\mu^*}{\mu_0} \le \frac{1 - v_p}{1 + \gamma v_p}, \tag{5}$$

where

$$\gamma = \frac{8 - 10\nu_0}{7 - 5\nu_0}. \tag{6}$$

The mentioned tendencies now require that the ratio

$$\frac{1 - v_p}{\frac{1 - v_p}{1 + \gamma v_p}} = 1 + \gamma v_p \tag{7}$$

is close to unity, with a slight tendency to increase with v_p. Indeed, it changes from about 1.1 to 1.2 (for a typical value of approximately $\nu_0 = 0.3$), as v_p increases from 0.1 to 0.2. Estimate (7) is relatively insensitive to the value of Poisson's ratio of the bulk material ν_0. It predicts that, for materials with lower ν_0, the slight increase in the yield strain may be more pronounced. Speifically, the parameter γ varies in the range of $2/3 \le \gamma \le 3/2$, where the lower bound corresponds to $\nu_0 = 0.5$ and the upper bound corresponds to $\nu_0 = -1$.

3. Concluding remarks. A relatively straightforward micromechanical explanation is given for the observation that the macroscopic strain at which yield occurs is approximately constant; more precisely, it has a slight tendency to grow as porosity increases. Furthermore, an earlier direct numerical analysis, conducted by the authors (Zohdi et al., 2002), confirms both the approximate constancy of the yield strain and its slight shift to the right We emphasize that these results are obtained under the assumption that the dense material (zero porosity) is elastic-perfectly plastic, i.e. it experiences no noticeable hardening. This is consistent with experimental evidence (Wang et al. 1996; Da Silva and Ramesh 1997a, 1997b; Kee et al. 1998) which suggests that, for materials exhibiting no appreciable work hardening and containing no more than about 20% volume fraction of pores, the macroscopic strain at which yield occurs is approximately constant, with a slight tendency to increase with pore volume fraction. The underlying concept - that the material is almost fully plasticized at yield - is further confirmed by our earlier direct microscale simulations (Zohdi et al., 2002) and studies of plastic percolation in GASAR materials, Kee et al. (1998).

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